MATH 70 FINAL EXAM REVIEW

1. Simplify each rational expression.

a) $\dfrac{y^2+y-20}{3xy^2-12xy-2y+8}$

b) $\dfrac{49m^2-n^2}{n^2-14mn+49m^2}$

c) $\dfrac{x^3-8}{x^2+2x-8} \div \dfrac{4x^2}{x^2+4x}$

2. State the domain. Express solution using set builder notation.

a) $f(x)=\dfrac{4x-3}{x^2-9}$

b) $f(x)=\dfrac{4x-20}{6x^2-x-5}$

c) $f(x)=\dfrac{3-x}{x^2+36}$

3. Divide. Use long division when appropriate.

a) $\dfrac{9m^3-54m^6+18m^2}{18m^2}$

b) $\dfrac{10x^2-17x+6}{5x-6}$

c) $\dfrac{y^4+2y^3-6y+3}{y^2+2}$

4. Given $f(x)=3x^2-8$

a. Find and simplify: $\dfrac{f(x)-f(h)}{x-h}$

b. Find and simplify: $\dfrac{f(x+h)-f(x)}{h}$

5. Find the LCD, add/subtract, then simplify.

a. LCD_____

$\dfrac{x}{x^2+5x+6}-\dfrac{2}{x^2+3x+2}$

b. LCD_____

$\dfrac{3}{y+4}-\dfrac{y}{y-1}+\dfrac{2y^2+3}{y^2+3y-4}$

6. Find the LCD, restriction(s), then solve.

LCD_____ Restriction(s) _____

a. $\dfrac{3}{x}+\dfrac{x}{4}=2$

LCD_____ Restriction(s) _____

b. $\dfrac{2x}{3}-\dfrac{8x-1}{4}=\dfrac{x}{6}$

LCD_____ Restriction(s) _____

c. $\dfrac{t-25}{t^2-t-12}+\dfrac{t}{t-4}=\dfrac{4}{t+3}$

LCD_____ Restriction(s) _____

d. $\dfrac{5y}{y+2}-\dfrac{6}{y}=1$

7. a. Solve by completing the square. Simplify.

$9x^2-12x+13=0$

b. Solve by using quadratic formula. Simplify.

$3x^2+6x+1=0$

8. An object projected upward with an initial velocity of 32 feet per second will rise and fall according to the equation $S(t) = 32t - 16t^2$, where S is its distance above the ground at time t. At what times will the object be 12 feet above the ground?

9. Graph $y = -(x+3)^2 + 4$ Label 3+ points. Include "Axis of Symmetry".

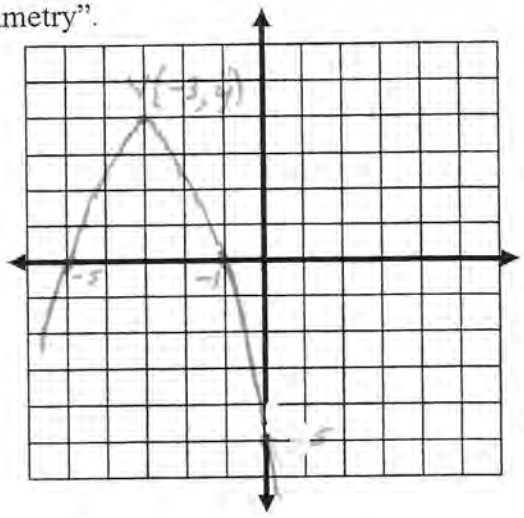
a) Compared to $f(x) = x^2$, we can conclude y:

i) has shifted _____ units UP DOWN Neither

ii) has shifted _____ units Left Right Neither

iii) opens: UP DOWN

x	y



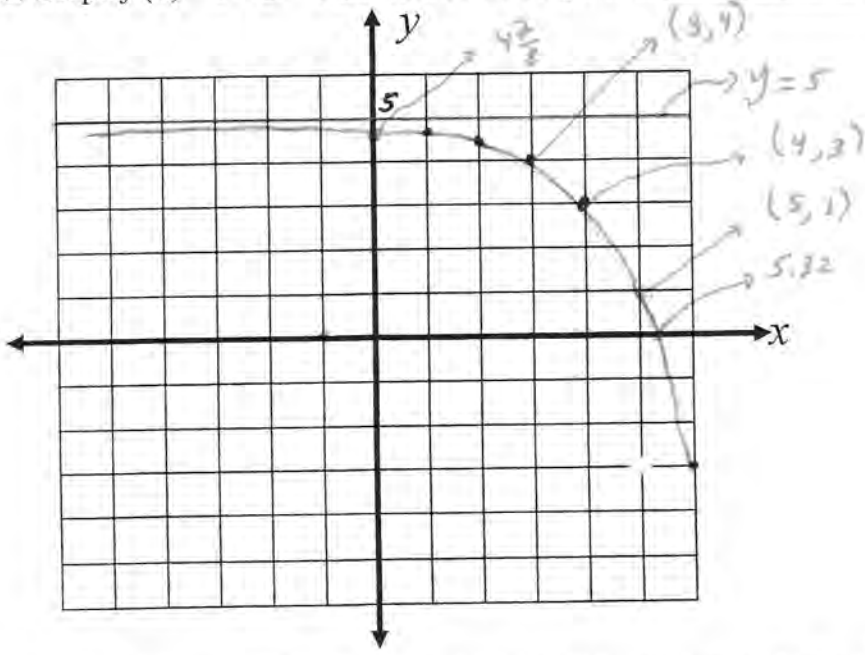
b) Find:

i) vertex (,) ii) x-intercepts: (,), (,); y-intercept (,)

c) Find:

i) Domain: Range: d) Axis of Symmetry (equation): _____

10. Graph $f(x) = 5 - 2^{x-3}$. **Label 3 or more points.** Draw horizontal asymptote.



b) Find:

Domain:

Range:

Y-int: (,)

Horizontal Asymptote (equation)

11. $A = P(1 + \frac{r}{n})^{nt}$; $A = Pe^{rt}$. a) $20,000 is invested at 3.8% for three years. Calculate the amount of money in the account (rounded to the nearest cent) if the money is compounded:

a) quarterly b) continuously

c) At the given interest rate, how long will it take for the $20,000 to reach $32,000, if compounded monthly?

12. Find the inverse of the function.

a. $f(x)=\frac{1}{2}x-7$

b. $f(x)=(x+5)^3-4$

13. a) Write in log form

$e^5 = y$

b) Write in exponential form

$\log_5 \frac{1}{125} = -3$

14. Evaluate.

a. $\log 0$

b. $\log_6(-36)$

c. $\log_7 49$

d. $\log_8 8$

e. $\log_5 5^8$

f. $\ln e^9$

g. $\log_4 \frac{1}{64}$

h. $\log\sqrt{100}$

i. $\log_2 \frac{1}{2}$

j. $-6\log_3 81$

k. $\log_2\left(\frac{1}{8}\right)+\log_2\left(\frac{1}{4}\right)$

l. $\log_5 375-\log_5 3$

m. $2\log_{12} 3+4\log_{12} 2$

15. Write as a single logarithm.

a. $\frac{1}{2}\log y-3\log x$

b. $(\log_7 a+5\log_7 b)-\log_7 c$

16. Express as the sum or difference of logarithms. Simplify.

a. $\log\left(x^2\sqrt{y}\right)$

b. $\log_5\left(\frac{125}{x^2}\right)$

17. Solve each equation.

a. $3^{5-2x}=\frac{1}{27}$

b. $8^{(4+2x)}=16$

c. $\log_5(2x-5)=2$

d. $\log_4 \frac{1}{64}=x$

e. $\log_b 49=2$

f. $\log_6 x+\log_6(x-5)=2$

18. Solve. Round answer to the nearest thousandth.

a. $6^{x-5}=97$

b. $104=8e^{-0.5t}$

Math 70 - Final Exam Review

1. a) $$\frac{y^2+y-20}{3xy^2-12xy-2y+8} = \frac{(y-4)(y+5)}{(y-4)(3xy-2)}$$

$$= \frac{y+5}{3xy-2}$$

$$y^2+y-20 = (y-4)(y+5)$$

$$3xy^2-12xy-2y+8 = (3xy^2-12xy)-(2y-8)$$

$$= 3xy(y-4) - 2(y-4)$$

$$= (y-4)(3xy-2)$$

b) $$\frac{49m^2-n^2}{n^2-14mn+49m^2} = \frac{(7m+n)(7m-n)}{(n-7m)(n-7m)}$$

$$= -\frac{7m+n}{n-7m} \text{ or } \frac{7m+n}{7m-n}$$

c) $$\frac{x^3-8}{x^2+2x-8} \div \frac{4x^2}{x^2+4x} =$$

$$= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+4)} \cdot \frac{x(x+4)}{4x^2}$$

$$= \frac{x^2+2x+4}{4x}$$

$M^2 4$

2a) $f(x) = \dfrac{4x-3}{x^2-9}$

$\{x \mid x \neq \pm 3\}$ $\quad x^2-9 = (x+3)(x-3)$

b) $f(x) = \dfrac{4x-20}{6x^2-x-5} = \dfrac{4(x-5)}{(6x+5)(x-1)}$

$\left\{x \mid x \neq 1, x \neq -\frac{5}{6}\right\}$

c) $f(x) = \dfrac{3-x}{x^2+36}$

Domain : All real numbers

3a) $\dfrac{9m^3 - 54m^6 + 18m^2}{18m^2} = \dfrac{9m^3}{18m^2} - \dfrac{54m^6}{18m^2} + \dfrac{18m^2}{18m^2}$

$= \qquad = \dfrac{m}{2} - 3m^4 + 1$

b)
$$\begin{array}{r} 2x-1 \\ 5x-6 \overline{)\ 10x^2-17x+6} \\ \underline{-10x^2 \pm 12x} \\ -5x+6 \\ \underline{\pm 5x \mp 6} \\ 0 \end{array}$$

$\therefore \dfrac{10x^2-17x+6}{5x-6} = 2x-1$

3c) $\dfrac{y^4+2y^3+0y^2-6y+3}{y^2+2} =$

$$y^2+2\overline{)\,y^4+2y^3+0y^2-6y+3}$$

$$y^2+2y-2$$

$$y^4+2y^2$$

$$2y^3-2y^2-6y$$

$$2y^3+4y$$

$$-2y^2-10y+3$$

$$-2y^2+4$$

$$-10y+7$$

$$y^2+2y-2+\frac{-10y+7}{y^2+2}$$

4. a) $f(x)=3x^2-8$

$$\frac{f(x)-f(h)}{x-h} = \frac{(3x^2-8)-(3h^2-8)}{x-h}$$

$$= \frac{3x^2-8-3h^2+8}{x-h}$$

$$= \frac{3(x^2-h^2)}{x-h} = \frac{3(x+h)(x-h)}{x-h}$$

$$= 3(x+h)$$

b) $$\frac{f(x+h)-f(x)}{h} = \frac{[3(x+h)^2-8]-[3x^2-8]}{h}$$

$$= \frac{3(x^2+2xh+h^2)-8-3x^2+8}{h}$$

$$= \frac{3x^2+6xh+3h^2-3x^2}{h} = \frac{3h(2x+h)}{h} = 3(2x+h)$$

5. $\dfrac{x}{x^2+5x+6} - \dfrac{2}{x^2+3x+2}$

a) $\dfrac{x}{(x+2)(x+3)} - \dfrac{2}{(x+2)(x+1)} =$

LCD: $(x+2)(x+3)(x+1)$

$= \dfrac{x}{(x+2)(x+3)} \cdot \dfrac{x+1}{x+1} - \dfrac{2}{(x+2)(x+1)} \cdot \dfrac{x+3}{x+3}$

$= \dfrac{x^2+x}{(x+2)(x+3)(x+1)} - \dfrac{2x+6}{(x+2)(x+1)(x+3)}$

$= \dfrac{(x^2+x)-(2x+3)}{(x+2)(x+3)(x+1)} = \dfrac{x^2+x-2x-3}{(x+1)(x+2)(x+3)}$

$= \dfrac{x^2-x-3}{(x+1)(x+2)(x+3)}$

5b) LCD: $(y+4)(y-1)$

$$\frac{3}{y+4}\cdot\frac{y-1}{y-1}-\frac{y(y+4)}{(y-1)(y+4)}+\frac{2y^2+3}{(y+4)(y-1)}$$

$$=\frac{3(y-1)}{(y+4)(y-1)}-\frac{y(y+4)}{(y-1)(y+4)}+\frac{2y^2+3}{(y+4)(y-1)}$$

$$=\frac{3y-3-y^2-4y+2y^2+3}{(y+4)(y-1)}=\frac{y^2-y}{(y+4)(y-1)}$$

$$=\frac{y(y-1)}{(y+4)(y-1)}=\frac{y}{y+4}$$

6. LCD: $4x$

a) Restriction: $x \neq 0$

$$\frac{3}{x} + \frac{x}{4} = 2$$

$$4x\left(\frac{3}{x} + \frac{x}{4}\right) = 4x(2)$$

$$12 + x^2 = 8x$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x = 2 \text{ or } x = 6$$

$$S = \{2, 6\}$$

6. b) LCD: 12

Restriction: None

$$\frac{2x}{3} - \frac{8x-1}{4} = \frac{x}{6}$$

$$12\left(\frac{2x}{3} - \frac{8x-1}{4}\right) = 12\left(\frac{x}{6}\right)$$

$$4(2x) - 3(8x-1) = 2x$$

$$8x - 24x + 3 = 2x$$

$$-16x - 2x = -3$$

$$16x + 2x = 3$$

$$18x = 3 \Rightarrow x = \frac{3}{18} = \frac{1}{6}$$

6c) $\frac{t-25}{t^2-t-12} + \frac{t}{t-4} = \frac{4}{t+3}$

$\frac{t-25}{(t+3)(t-4)} + \frac{t}{t-4} = \frac{4}{t+3}$

LCD: $(t+3)(t-4)$

Restriction: $t \neq -3$, $t \neq 4$

$(t+3)(t-4)\left[\frac{t-25}{(t+3)(t-4)} + \frac{t}{t-4}\right] = (t+3)(t-4)\left(\frac{4}{t+3}\right)$

$t-25 + (t+3)t = 4(t+3)$

$t-25+t^2+3t = 4t+12$

$t^2+4t-4t-25-12 = 0$

$t^2-13=0$

$t^2=13$

$t = \pm\sqrt{13}$

6d) LCD: $y(y+2)$

Restriction: $y \neq 0$, $y \neq -2$

$y(y+2)\left(\frac{5y}{y+2} - \frac{6}{y}\right) = y(y+2)(1)$

$y(5y) - 6(y+2) = y(y+2)$

$5y^2 - 6y - 12 - y^2 - 2y = 0$

$4y^2 - 8y - 12 = 0$

$y^2 - 2y - 3 = 0$

$(y+1)(y-3) = 0$

$y = -1$, or $y = 3$

$S = \{-1, 3\}$

7a. $9x^2 - 12x + 13 = 0$

$(9x^2 - 12x + 4) + 9 = 0$

$(3x - 2)^2 = -9$

$3x - 2 = \sqrt{-9} = \pm 3i$

$3x = \pm 3i + 2$

$x = \frac{2}{3} \pm i$

I THINK WHAT SHE MEANT TO ASK WAS THIS:

$\boxed{9x^2 - 12x - 5 = 0}$

THIS WORKS OUT NICER, ALTHOUGH IT'S A TOTALLY DIFFERENT PROBLEM. I HAVE PROVIDED A SOLUTION IF YOU'D LIKE TO DO IT FOR PRACTICE

$9x^2 - 12 - 5 = 0$

$(9x^2 - 12 + 4) - 9 = 0$

$(3x - 2)^2 = 9$

$3x - 2 = \pm 3$

$3x - 2 = 3$ | $3x - 2 = -3$

$3x = 5$ | $3x = -1$

$x = \frac{5}{3}$ | $x = -\frac{1}{3}$

7b $3x^2 + 6x + 1 = 0$, $a = 3, b = 6, c = 1$

$\frac{-6 \pm \sqrt{36 - 12}}{6} = \frac{-6 \pm \sqrt{24}}{6} = \frac{-6 \pm 2\sqrt{6}}{6} = \frac{-3 \pm \sqrt{6}}{3}$

$= \left\{-1 + \frac{\sqrt{6}}{3}, -1 - \frac{\sqrt{6}}{3}\right\}$

8. $s(t) = 32t - 16t^2 = 12$

$32t - 16t^2 - 12 = 0$

$8t - 4t^2 - 3 = 0$

$-4t^2 + 8t - 3 = 0$

$4t^2 - 8t + 3 = 0$

$(2t - 1)(2t - 3) = 0$

$t = \frac{1}{2}$ sec, $t = \frac{3}{2} = 1\frac{1}{2}$ sec.

9. $y = -(x+3)^2 + 4$

a) i) y has shifted 4 units up.

ii) y has shifted 3 units to the left

iii) opens: Down

b) i) Vertex: $(-3, 4)$ Graph: See review sheet

ii) x-intercept:

Let $y = 0$. Then $-(x+3)^2 + 4 = 0$

$(x+3)^2 = 4 \Rightarrow x + 3 = \pm\sqrt{4}$

$x + 3 = \pm 2 \Rightarrow x = -3 \pm 2$

$x = -5$ or $x = -1 \Rightarrow (-5, 0), (-1, 0)$

y-intercept: Let $x = 0$. Then

$y = -(0+3)^2 + 4 =$

$= -9 + 4$

$y = -5 \Rightarrow (0, -5)$

9c) Domain: All real numbers

Range: $\{y : y \leq 4\}$

d) Axis of symmetry is:

$$x + 3 = 0$$
$$x = -3$$

10. $f(x) = 5 - 2^{x-3}$

a) See graph paper – Review sheet

b) Domain: All real numbers

x	y
1	$4\frac{3}{4}$
2	$4\frac{1}{2}$
3	4
4	3
5	1
6	-3

Range: $\{y \mid y < 5\}$

y-intercept: Let $x = 0$. Then

$$y = 5 - 2^{0-3} = 5 - \frac{1}{2^3}$$

$$= 5 - \frac{1}{8} = 4\frac{7}{8}$$

Horizontal asymptote: $y = 5$

11a) $A = p\left(1+\frac{r}{n}\right)^{nt}$

$= 20{,}000\left(1+\frac{.038}{4}\right)^{4(3)}$

$= 20{,}000(1+.0095)^{12} = 20{,}000(1.0095)^{12}$

$= 20{,}000(1.1201492) = \$22402.98$

b) $A = pe^{rt}$

$= 20{,}000\, e^{.038(3)} = 20{,}000\, e^{.114}$

$= 20{,}000(1.12075) = \$22415.04$

c) $A = p\left(1+\frac{r}{n}\right)^{nt}$

$32{,}000 = 20{,}000\left(1+\frac{.038}{12}\right)^{12t}$

$32000 = 20{,}000(1.00317)^{12t}$

$1.6 = 1.00317^{12t}$

$\log 1.6 = \log 1.00317^{12t} = 12t \log 1.00317$

$\frac{\log 1.6}{\log 1.00317} = 12t$

$12t = \frac{.2041199}{.001374\ldots} = 148.5048$

$t = 12.375 \approx 12.38$ years

12.

a) $f(x) = \frac{1}{2}x - 7$

$y = \frac{1}{2}x - 7$

$x = \frac{1}{2}y - 7$

$x + 7 = \frac{1}{2}y$

$2x + 14 = y$

$y = 2x + 14$

$f^{-1}(x) = 2x + 14$

b) $f(x) = (x+5)^3 - 4$

$y = (x+5)^3 - 4$

$x = (y+5)^3 - 4$

$x + 4 = (y+5)^3$

$\left[(y+5)^3\right]^{\frac{1}{3}} = (x+4)^{\frac{1}{3}}$

$y + 5 = \sqrt[3]{x+4}$

$y = \sqrt[3]{x+4} - 5$

$f^{-1}(x) = \sqrt[3]{x+4} - 5$

13a) $e^5 = y$

$\log_e y = 5$

$\ln y = 5$

b) $\log_5 \frac{1}{125} = -3$

$5^{-3} = \frac{1}{125}$

14 a) $\log 0$

$\log_b N = E$ is defined when $b > 0$ and $b \neq 1$ and $N > 0$

$\log 0$ is undefined.

b) $\log_6 (-36) = x$

$6^x = -36 = -6^2$

Not defined (See part a)

c) $\log_7 49 = \log_7 7^2 = 2$

d) $\log_8 8^1 = 1$

e) $\log_5 5^8 = 8$

14.

f) $\ln e^9 = 9$

g) $\log_4 \frac{1}{64} = \log_4 \frac{1}{4^3} = \log_4 4^{-3}$

$= -3$

h) $\log \sqrt{100} = \log 100^{\frac{1}{2}} = 1$

i) $\log_2 \frac{1}{2} = \log_2 2^{-1} = -1$

j) $-6 \log_3 81 = -3 \log_3 3^4$

$= -3(4) = -12$

k) $\log_2 \left(\frac{1}{8}\right) + \log_2 \left(\frac{1}{4}\right) = \log_2 2^{-3} + \log_2 2^{-2}$

$= -3 + (-2) = -5$

l) $\log_5 375 - \log_5 3 = \frac{\log 375}{\log 5} - \frac{\log 3}{\log 5}$

$= \frac{2.5740}{0.69897} - \frac{0.4772}{0.69897}$

$= 3.6826 - 0.6826$

$= 3$

ANOTHER WAY: $\log_5 \underbrace{3 \times 125}_{\text{SINCE } 375 = 3 \times 125} - \log_5 3 = \overbrace{\log_5 3 + \log_5 125}^{\text{SINCE } \log ab = \log a + \log b} - \log_5 3$

$= \log_5 125$

$= 3 \quad (\text{SINCE } 5^3 = 125)$

15. a) $\frac{1}{2}\log y - 3\log x = \log y^{\frac{1}{2}} - \log x^3$

$= \log \frac{\sqrt{y}}{x^3}$

b) $\left(\log_7 a + 5\log_7 b\right) - \log_7 c = \log_7 a + \log_7 b^5 - \log_7 c$

$= \log_7 \frac{ab^5}{c}$

16 a) $\log\left(x^2\sqrt{y}\right) = \log x^2 + \log \sqrt{y}$

$= 2\log x + \frac{1}{2}\log y$

b) $\log_5 \frac{125}{x^2} = \log_5 125 - \log_5 x^2$

$= \log_5 5^3 - 2\log_5 x$

$= 3 - 2\log_5 x$

17. a) $3^{5-2x} = \frac{1}{27}$

$3^{5-2x} = 3^{-3}$

$5 - 2x = -3$

$5 + 3 = 2x$

$2x = 8$

$x = 4$

b) $8^{4+2x} = 16$

$\left(2^3\right)^{4+2x} = 2^4$

$2^{12+6x} = 2^4$

$12 + 6x = 4$

$6x = 4 - 12$

$6x = -8$

$x = -\frac{8}{6} = -\frac{4}{3}$

c) $\log_5 (2x - 5) = 2$

$5^2 = 2x - 5$

$25 = 2x - 5$

$30 = 2x$

$x = 15$

d) $\log_4 \frac{1}{64} = x$

$4^x = \frac{1}{64} = \frac{1}{4^3} = 4^{-3}$

$x = -3$

17.

e) $\log_b 49 = 2$

$b^2 = 49 = 7^2$

$b = 7$

f) $\log_6 x + \log_6 (x-5) = 2$

$\log_6 x(x-5) = 2$

$6^2 = x(x-5)$

$0 = x^2 - 5x - 36$

$(x+4)(x-9) = 0$

$x = -4$ Reject

$x = 9$

18. $6^{x-5} = 97$

a) $\log 6^{x-5} = \log 97$

$(x-5)\log 6 = \log 97$

$x - 5 = \frac{\log 97}{\log 6} = \frac{1.9868}{0.7782}$

$x - 5 = 2.5532$

$x = 7.5532 \approx 7.55$

b) $104 = 8e^{-0.5t}$

$\frac{104}{8} = \frac{8e^{-0.5t}}{8}$

$13 = e^{-0.5t}$

$\ln 13 = \ln e^{-0.5t}$

$\frac{2.5649}{-0.5} = \frac{-0.5t}{-0.5}$

$t = -5.1299$

$t = -5.13$